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Permutations and Combinations (Miscellaneous Exercise)

Q. 1 A Committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the Committee consists of atmost 3 girls?

Soln: Here, we see that to select atmost 3 girls i.e. there are following four possibilities

① 9C_4 ways ② 9C_3 ways ③ 9C_2 ways and ④ 9C_1 ways

or ~~① 9C_6 ways ② 9C_5 ways ③ 9C_4 ways ④ 9C_3 ways~~

Hence, total number of ways

$$\begin{aligned} &= ({}^9C_4 \times {}^4C_0) + ({}^9C_3 \times {}^4C_1) + ({}^9C_2 \times {}^4C_2) + ({}^9C_1 \times {}^4C_3) \\ &= \left(\frac{9!}{4!5!} \times \frac{4!}{0!4!} \right) + \left(\frac{9!}{3!6!} \times \frac{4!}{1!3!} \right) + \left(\frac{9!}{2!7!} \times \frac{4!}{2!2!} \right) + \left(\frac{9!}{1!8!} \times \frac{4!}{3!1!} \right) \\ &= \left(\frac{9!}{4!5!} \right) + \left(\frac{9!}{3!6!} \times \frac{4!}{1!3!} \right) + \left(\frac{9!}{2!7!} \times \frac{4!}{2!2!} \right) + \left(\frac{9!}{1!8!} \times \frac{4!}{3!1!} \right) \end{aligned}$$

$$\begin{aligned} &= \left(\frac{9 \times 8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} \right) + \left(\frac{9 \times 8 \times 7 \times 6 \times 5}{3 \times 2 \times 1 \times 2 \times 1} \right) + \left(\frac{9 \times 8 \times 7 \times 6 \times 5}{2 \times 1 \times 3 \times 2 \times 1 \times 1 \times 1} \right) + \left(\frac{9 \times 8 \times 7 \times 6 \times 5}{1 \times 7 \times 2 \times 1 \times 1} \times \frac{1}{1} \right) \end{aligned}$$

$$\begin{aligned} &= \left(\frac{9 \times 8 \times 7 \times 6}{6} \right) + \left(\frac{9 \times 8 \times 7 \times 6}{4} \right) + \left(\frac{9 \times 8 \times 7}{2} \times 4 \right) + \left(\frac{9 \times 8}{2} \right) \end{aligned}$$

$$= 72 \times 7 + (9 \times 2 \times 7 \times 6) + (3 \times 4 \times 4 \times 7) + (9 \times 4)$$

$$= 72 \times 7 + 756 + 48 \times 7 + 36$$

$$= 504 + 756 + 336 + 36$$

$$= 1632 \text{ ways.}$$

$$\begin{array}{r} 504 \\ 756 \\ 336 \\ 36 \\ \hline 1632 \end{array}$$

Permutations and Combinations (Miscellaneous Exercise)

Q.2. If the different permutations of all the letters of the word "EXAMINATION" are listed as in a dictionary, how many words are there in this list before the first word starting with E?

Soln: The letters of given word are A, A, I, E, M, N, N, O, T, X i.e. Word starting with A are formed with the letters 2 I's, 2 N's, A, E, X, M, T, O.

Total No. of letters = 10.

Hence, number of words formed by these letters

$$= \frac{10!}{2!2!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 2 \times 1}$$

$$= 45 \times 56 \times 30 \times 12 = 907200 \text{ ways}$$

Q.3. How many 6 digit numbers can be formed from the digits 0, 1, 3, 5, 7 and 9 which are divisible by 10 and no digit is repeated?

Soln: $\square \square \square \square \square \square$

Number of ways to fill the unit place = 1

(i.e. 0 in unit place) and remaining 5 numbers (1, 3, 5, 7, 9) can be arranged in 120 ways.

Hence, by Fundamental Principle of Counting, total number of ways

$$= 120 \times 1 = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

Q.4. Determine the number of 5 card combinations out of a deck of 52 cards if each selection of 5 cards has exactly one King.

Soln: Since, 1 King out of 4 Kings can be selected in 4C_1 ways. Remaining 4 cards out of remaining 48 cards can be selected in ${}^{48}C_4$ ways. Hence, by Fundamental Principle of Counting,

$$\text{total number of ways} = {}^4C_1 \times {}^{48}C_4 = 688320$$